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NOTES

A Nonlinear Maxwell Model to Explain the Initial Fall in Dynamic Modulus at Constant Static Strain

INTRODUCTION

It is well known that the viscoelastic properties of textile fibers at finite strains cannot be explained in terms of idealized linear viscoelastic models. One of the reasons for this is the dependence of the modulus on the static strain in the fiber, as demonstrated by the fact that the dynamic modulus increases with the static strain applied to the fiber. Although the dynamic modulus remains virtually constant at constant static strain, at least two workers[1,2] have observed a slight fall in the dynamic modulus at constant static strain. Some attempts have been made[3] to include the stiffening of fiber in the viscoelastic theory. This theory cannot explain the initial fall in modulus, because whatever may be the relation between the stiffness (dynamic modulus) and the static strain, as long as the static strain is kept constant the stiffness or the dynamic modulus should remain constant.

NONLINEAR MODEL

The following model is suggested to explain the initial fall in the dynamic modulus at constant strain.

Here it is assumed that in a generalized Maxwell model the spring constants vary linearly with the strain on the spring, and the viscosity constant η is independent of the strain.

QUALITATIVE EXPLANATION OF THE BEHAVIOR OF THE MODEL

In stress relaxation experiments, the instantaneous strain applied is taken up by the springs alone; and as time passes, the springs contract. This results in a decrease in the dynamic modulus (the real part of the complex modulus) according to the assumption stated above.

In practice, the modulus is found to stabilize virtually after some time. This should be thoroughly verified experimentally by taking observations for a sufficiently long period. The modulus may be found either to decrease infinitely, as in stress relaxation, or it may level off. Even if it levels off, the present model can be suitably modified to explain such behavior.

MATHEMATICAL FORMULATION

The real part of the complex modulus of the model can be determined as follows:

Let x, y and z be respectively the extensions in the spring, the dashpot, and the fiber:

$$\therefore z = x + y \tag{1}$$

Let F be the force in the fiber, and $K = dF/dx$ and η be the spring constant and the viscosity constant, respectively. K increases linearly with x:

$$\therefore K = K_0 + \lambda \cdot x = \frac{dF}{dx} \tag{2}$$

where K_0 and λ are constants.

$$\therefore F = K_0 \cdot x + \frac{\lambda}{2} x^2 + \text{constant}$$

when $x = 0, F = 0$; so constant $= 0$;

$$\therefore F = K_0 \cdot x + \frac{\lambda}{2} x^2 \tag{3}$$

$$F = \eta \frac{dy}{dt}. \tag{4}$$

In stress relaxation experiments, the strain $z = \epsilon$ is applied instantaneously and maintained constant throughout the experiment. Initially, the entire strain is taken up by the spring alone.

$$\left.\begin{aligned} \therefore \text{ when } t &= 0 \\ x = z &= \epsilon \\ \text{and } y &= 0 \end{aligned}\right\} \tag{5}$$

$$F = \eta \frac{dy}{dt} = \eta \frac{d(z - x)}{dt} = -\eta \frac{dx}{dt}$$

$$\therefore F = -\eta \frac{dx}{dt} = K_0 \cdot x + \frac{\lambda}{2} x^2$$

or

$$\frac{dx}{K_0 \cdot x + \frac{\lambda}{2} x^2} + \frac{dt}{\eta} = 0 \tag{6}$$

The solution of this differential equation is

$$\frac{1}{K_0} \ln \frac{x}{\frac{\lambda}{2} x + K_0} + \frac{t}{\eta} = \text{constant}.$$

When $t = 0, x = \epsilon$;

$$\therefore \text{constant} = \frac{1}{K_0} \ln \frac{\epsilon}{\frac{\lambda}{2} \epsilon + K_0} = \frac{1}{K_0} \ln \beta \text{ (say)}$$

$$\therefore \frac{x}{\frac{\lambda}{2} x + K_0} = \beta e^{-t/\eta/K_0}$$

or

$$x = \frac{K_0 \beta e^{-t/\tau}}{1 - \frac{\lambda}{2} \beta e^{-t/\tau}} \tag{7}$$

where $\tau = \eta/K_0$.

ILLUSTRATIVE CALCULATIONS

To illustrate the behavior of such a model, a generalized Maxwell model with the following parameters was chosen:

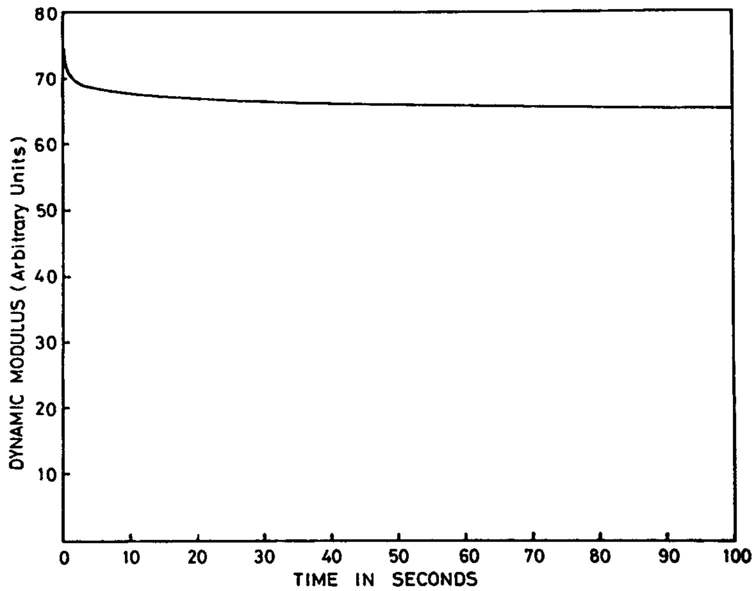

Fig. 1. Initial fall in dynamic modulus as a function of time.

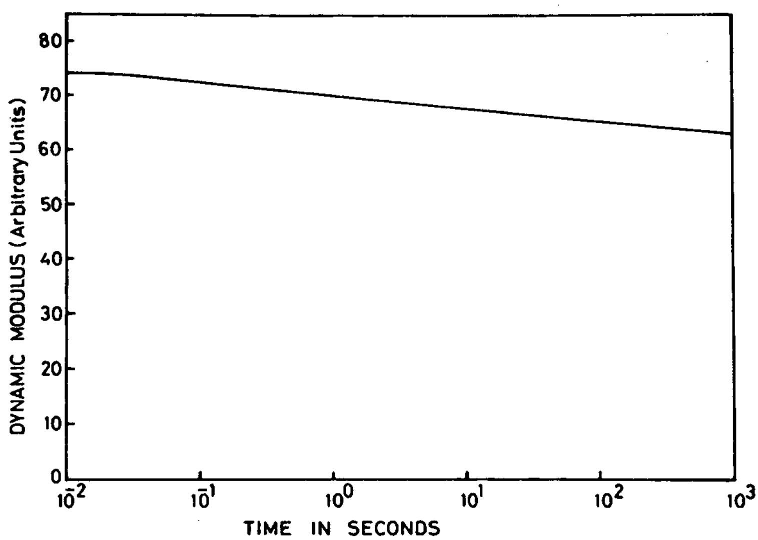

Fig. 2. Fall in dynamic modulus as a function of time.

number of Maxwell elements = 6

$$K_{01} = K_{02} = K_{03} = K_{04} = K_{05} = K_{06} = 10$$

$$\tau_1 = 0.1,\ \tau_2 = 1.0,\ \tau_3 = 10,\ \tau_4 = 100,\ \tau_5 = 1{,}000,\ \tau_6 = 10{,}000 \text{ sec}$$

$$\lambda = 30;\ \epsilon = \text{applied strain} = 0.08\ (8\%).$$

Using the above-mentioned numerical constants and eqs. (2) and (7), the decaying modulus was calculated as a function of time. The initial portion of this curve up to 100 sec is illustrated in Figure 1. This curve is qualitatively similar to the curve given by de Vries.[2] When the same curve is plotted with a logarithmic time scale, it is seen that the modulus falls even at 1000 sec (Fig. 2).

CONCLUSIONS

A nonlinear Maxwell model is proposed in which the spring constants increase linearly with the strain on the spring, the viscosity constants of the dashpots are assumed to be independent of strain. This model can explain the increase in the dynamic modulus with static strain and also the initial fall in the dynamic modulus at constant static strain.

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